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Wave function correlations on the ballistic scale: Exploring quantum chaos by quantum disorder

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We study the statistics of wave functions in a ballistic chaotic system. The statistical ensemble is generated by adding weak smooth disorder. The conjecture of Gaussian fluctuations of wave functions put forward by Berry [J. Phys. A **10**, 2083 (1977)] and generalized by Hortikar and Srednicki [Phys. Rev. Lett. **80**, 1646 (1998); Phys. Rev. E **57**, 7313 (1998)] is proven to hold on sufficiently short distances, while it is found to be strongly violated on larger scales. This also resolves the conflict between the above conjecture and the wave function normalization. The method is further used to study ballistic correlations of wave functions in a random magnetic field.

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INTRODUCTION

Understanding of statistical properties of eigenfunctions of a quantum system whose classical counterpart is chaotic and their relation to the underlying classical dynamics is one of the key problems studied in the field of quantum chaos. Among various applications, wave function correlations are important for statistics of electron transport through quantum dots, see Refs. [1,2] and references therein. It was conjectured by Berry [3] that an eigenfunction of a classically chaotic system ("billiard") can be represented as a random superposition of plane waves with fixed absolute value k of the wave vector (determined by the energy $k^2/2m=E$, where m is the mass and we set $\hbar=1$). This implies Gaussian statistics of the eigenfunction amplitude $\psi(\mathbf{r})$,

$$\mathcal{P}\{\psi\}\propto\exp\left[-\frac{\beta}{2}\int d^2\mathbf{r}d^2\mathbf{r}'\,\psi^*(\mathbf{r})C^{-1}(\mathbf{r},\mathbf{r}')\psi(\mathbf{r}')\right], \quad (1)$$

determined solely by the correlation function (we consider a two-dimensional system)

$$C(\mathbf{r}_1,\mathbf{r}_2)\equiv\langle\psi^*(\mathbf{r}_1)\psi(\mathbf{r}_2)\rangle=J_0(k|\mathbf{r}_1-\mathbf{r}_2|)/V. \quad (2)$$

Here $\beta=1$ or 2 for a system with preserved (respectively, broken) time reversal symmetry, V is the system area, and J_0 the Bessel function. For definiteness, we will consider the case $\beta=2$ below; generalization to systems with $\beta=1$ is straightforward.

Hortikar and Srednicki [4] noticed that Eqs. (1), and (2), which do not depend on any details of the dynamics, may only be valid for sufficiently small spatial separation. They generalized Berry's hypothesis and conjectured the Gaussian statistics (1) with a more general, system-specific kernel $C(\mathbf{r}_1,\mathbf{r}_2)$ replacing Eq. (2),

$$C(\mathbf{r}_1,\mathbf{r}_2)=\frac{\operatorname{Im}G_{\rm sc}(\mathbf{r}_1,\mathbf{r}_2)}{\int d\mathbf{r}\,\operatorname{Im}G_{\rm sc}(\mathbf{r},\mathbf{r})}, \quad (3)$$

where $G_{\rm sc}$ is a semiclassical Green's function [4]. This proposal was supported by the observation [5] that the result obtained in Ref. [6] for two-point correlations in a diffusive system is consistent with the Gaussian statistics.

The conjecture (1), (3), while physically very appealing, obviously requires a formal derivation. Furthermore, when taken literally, this conjecture contradicts the wave function normalization,

$$\int d\mathbf{r}[\langle|\psi^2(\mathbf{r})\psi^2(\mathbf{r}')|\rangle-\langle|\psi^2(\mathbf{r})|\rangle\langle|\psi^2(\mathbf{r}')|\rangle]=0, \quad (4)$$

since the integrand is equal to $C^2(\mathbf{r},\mathbf{r}')>0$ according to Eq. (1). Therefore, limits of validity of this conjecture have to be understood. All this points to a need of a systematic study of wave function statistics in ballistic systems, which is the aim of the present paper.

EIGENFUNCTION STATISTICS IN A BALLISTIC SYSTEM

In order to speak about the wave function statistics $\mathcal{P}\{\psi(\mathbf{r})\}$, one should first define an ensemble over which the averaging goes. Such an ensemble can be generated [7] by adding to a system under consideration a random potential $U(\mathbf{r})$ characterized by a correlation function $W(\mathbf{r}-\mathbf{r}')=\langle U(\mathbf{r})U(\mathbf{r}')\rangle$ with a correlation length d. Parameters of this random potential are assumed to satisfy $k^{-1}\ll d\ll l_s\ll L\ll l_{\rm tr}$, where l_s ($l_{\rm tr}$) is the single-particle (respectively, transport) mean free path, and L is the characteristic size of the system. The condition $l_{\rm tr}\gg L$ follows from the requirement that the additional disorder does not influence the classical dynamics of the system, while the inequality $l_s\ll L$ guarantees that the ensemble of quantum systems is large enough to produce meaningful result. Note that the potential is smooth, $kd\gg1$, since $l_{\rm tr}/l_s\sim(kd)^2$. On the technical side, introducing the additional random potential allows us to apply, with a proper generalization, methods developed earlier for diffusive systems (see Ref. [8] for a review).

After the ensemble averaging, the problem is described by

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a ballistic σ model of a supermatrix field $Q(\mathbf{r},\mathbf{n})$ with the action [9–11,7]

$$S[Q]=\mathrm{Str}\ln\left[E-\hat{H}+i\eta\Lambda-\frac{i}{2}\int d\mathbf{n}'Q(\mathbf{r},\mathbf{n}')w(\mathbf{n},\mathbf{n}')\right]-\frac{\pi\nu}{4}\int d^2\mathbf{r}d\mathbf{n}d\mathbf{n}'\,\mathrm{Str}Q(\mathbf{r},\mathbf{n})w(\mathbf{n},\mathbf{n}')Q(\mathbf{r},\mathbf{n}'),\tag{5}$$

where $\hat{H}$ is the Hamiltonian of the system (without disorder), ν is the density of states, $w(\mathbf{n},\mathbf{n}')=2\pi\nu W(k|\mathbf{n}-\mathbf{n}'|)$ is the scattering cross section by the random potential, and $\mathbf{n}$ is a unit vector characterizing the direction of velocity on the energy surface. On the scales $\gg l_s$ Eq. (5) reduces to the form proposed in Refs. [12,13]. The two-point correlation function of the wave function intensities is expressed in this approach as [8,7]

$$\langle|\psi^2(\mathbf{r}_1)\psi^2(\mathbf{r}_2)|\rangle=\lim_{\eta\to 0}\frac{\eta\Delta}{\pi}\langle[G_{11}(\mathbf{r}_1,\mathbf{r}_1)G_{22}(\mathbf{r}_2,\mathbf{r}_2)+G_{12}(\mathbf{r}_1,\mathbf{r}_2)G_{21}(\mathbf{r}_2,\mathbf{r}_1)]\rangle_{S[Q]},\tag{6}$$

where Δ is the mean level spacing, $\hat{G}$ is the Green's function in the field Q,

$$\hat{G}=\left[E-\hat{H}+i\eta\Lambda-\frac{i}{2}\int d\mathbf{n}'Q(\mathbf{r},\mathbf{n}')w(\mathbf{n},\mathbf{n}')\right]^{-1},\tag{7}$$

and the subscripts 1, 2 refer to the advanced-retarded decomposition (the boson-boson components being implied). We first evaluate Eq. (6) in the zero-mode approximation, $Q(\mathbf{r})=Q_0$. The Green's function (7) is given in the leading order by

$$G_0(\mathbf{r},\mathbf{r}')=i\,\mathrm{Im}\,G^R(\mathbf{r},\mathbf{r}')Q_0+\mathrm{Re}\,G^R(\mathbf{r},\mathbf{r}'),\tag{8}$$

$$G^R(\mathbf{r},\mathbf{r}')=\langle\mathbf{r}|(E-\hat{H}+i/2\tau_s)^{-1}|\mathbf{r}'\rangle.\tag{9}$$

If the points $\mathbf{r},\mathbf{r}'$ are separated by a distance $\gg l_s$ from the billiard boundary, Eq. (9) reduces to $\mathrm{Im}\,G^R(\mathbf{r},\mathbf{r}')=-\pi\nu J_0(k|\mathbf{r}-\mathbf{r}'|)e^{-|\mathbf{r}-\mathbf{r}'|/2l_s}$. Substituting Eq. (8) in Eq. (6) and expanding the action (5) up to the linear-in-η term, $S[Q]\simeq\pi\nu\eta V\,\mathrm{Str}\,Q_0\Lambda$, one finds, in full analogy with the case of diffusive systems,

$$V^2\langle|\psi^2(\mathbf{r}_1)\psi^2(\mathbf{r}_2)|\rangle\simeq 1+k_q(\mathbf{r}_1,\mathbf{r}_2),\tag{10}$$

$$k_q(\mathbf{r},\mathbf{r}')=\mathrm{Im}\,G^R(\mathbf{r},\mathbf{r}')\,\mathrm{Im}\,G^R(\mathbf{r}',\mathbf{r})/(\pi\nu)^2,\tag{11}$$

with the two contributions on the rhs of Eq. (10) originating from the terms $\langle G_{11}G_{22}\rangle$ and $\langle G_{12}G_{21}\rangle$ in Eq. (6), respectively. The result (10), corresponding exactly to the conjecture (1), (3) of the Gaussian statistics, is in conflict with the wave function normalization, as explained above.

To resolve this problem, we evaluate the term $\langle G_{11}G_{22}\rangle$ more accurately by expanding the Green's function (7) to the order η and the action (5) to the order η^2. While these terms (usually neglected in the σ-model calculations) are of the next order in η and may be naively thought to vanish in the limit $\eta\to 0$ performed in Eq. (6), this is not so, since $Q_0\propto\eta^{-1}$. As a result, we get in the zero-mode approximation

$$V^2\langle|\psi^2(\mathbf{r}_1)\psi^2(\mathbf{r}_2)|\rangle_{\mathrm{ZM}}-1=k_q(\mathbf{r}_1,\mathbf{r}_2)-\overline{k_q}(\mathbf{r}_1)-\overline{k_q}(\mathbf{r}_2)+\bar{\bar{k}}_q\tag{12}$$

(terms of still higher orders in η produce corrections small in the parameter $\Delta\tau_s\ll 1$), where

$$\overline{k_q}(\mathbf{r})=V^{-1}\int d^2\mathbf{r}'k_q(\mathbf{r},\mathbf{r}'),$$

$$\bar{\bar{k}}_q=V^{-2}\int d^2\mathbf{r}d^2\mathbf{r}'k_q(\mathbf{r},\mathbf{r}').\tag{13}$$

The contribution of nonzero modes is found to be [7]

$$V^2\langle|\psi^2(\mathbf{r}_1)\psi^2(\mathbf{r}_2)|\rangle_{\mathrm{NZM}}=\tilde{\Pi}_B(\mathbf{r}_1,\mathbf{r}_2),\tag{14}$$

where $\tilde{\Pi}_B(\mathbf{r}_1,\mathbf{r}_2)=\Pi_B(\mathbf{r}_1,\mathbf{r}_2)-\Pi_B^{(0)}(\mathbf{r}_1,\mathbf{r}_2)$ describes the (integrated over direction of velocity) probability of classical propagation from $\mathbf{r}_1$ to $\mathbf{r}_2$,

$$\Pi_B(\mathbf{r}_1,\mathbf{r}_2)=\int\int d\mathbf{n}_1d\mathbf{n}_2\mathcal{D}(\mathbf{r}_1\mathbf{n}_1,\mathbf{r}_2\mathbf{n}_2),$$

$$\hat{\mathcal{L}}\mathcal{D}=(\pi\nu)^{-1}[\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{n}_1-\mathbf{n}_2)-V^{-1}],\tag{15}$$

with the contribution $\Pi_B^{(0)}(\mathbf{r}_1,\mathbf{r}_2)$ of direct propagation (before the first event of disorder scattering) excluded. The symbol $\hat{\mathcal{L}}$ in Eq. (15) denotes the Liouville operator characterizing the classical motion [14].

We analyze now the total result given by the sum of Eqs. (12) and (14). First of all, we stress that it satisfies exactly the constraint (4) of wave function normalization. Next, we consider sufficiently short distances, $|\mathbf{r}_1-\mathbf{r}_2|\ll l_s$. In this case the correlation function is dominated by the first term in the rhs of Eq. (12), returning us to the result (10). Furthermore, we can generalize this result to higher correlation functions,

$$\langle\psi^*(\mathbf{r}_1)\psi(\mathbf{r}_1')\dots\psi^*(\mathbf{r}_n)\psi(\mathbf{r}_n')\rangle=-\frac{1}{2V(n-1)!}\lim_{\eta\to 0}(2\pi\nu\eta)^{n-1}\times\left\langle\sum_\sigma\prod_{i=1}^n\frac{1}{\pi\nu}G_{p_ip_{\sigma(i)}}(\mathbf{r}_i,\mathbf{r}_{\sigma(i)}')\right\rangle_{S[Q]},$$

where the summation goes over all permutations σ of the set $\{1,2,...,n\}$, $p_i=1$ for $i=1,...,n-1$, and $p_n=2$. If all the points $\mathbf{r}_i,\mathbf{r}_i'$ are within a distance $\ll l_s$ from each other, the leading contribution to this correlation function is given by the zero-mode approximation with higher-order terms in η neglected [i.e., by the same approximation which leads to Eq. (10)], yielding

$$V^n\langle\psi^*(\mathbf{r}_1)\psi(\mathbf{r}_1')\ldots\psi^*(\mathbf{r}_n)\psi(\mathbf{r}_n')\rangle=\sum_\sigma\prod_{i=1}^n f_F(\mathbf{r}_i,\mathbf{r}'_{\sigma(i)}),$$

$$f_F(\mathbf{r},\mathbf{r}')=-\operatorname{Im}G^R(\mathbf{r},\mathbf{r}')/(\pi\nu). \tag{16}$$

This result is identical to the statement (1) of the Gaussian statistics of eigenfunctions. We have thus proven that the conjecture of Refs. [3,4] holds within a spatial region of an extension $\ll l_s$, with the kernel $C(\mathbf{r}_1,\mathbf{r}_2)$ given by Eqs. (16) and (9).

We turn now to the behavior of the correlator $\langle|\psi^2(\mathbf{r}_1)\psi^2(\mathbf{r}_2)|\rangle$ at larger separations $|\mathbf{r}_1-\mathbf{r}_2|\gg l_s$. In this situation, the correlations are dominated by the contribution (14) of nonzero modes. Let us further note that the smooth part of the zero-mode contribution (12) (i.e., with Friedel-type oscillations on the scale of the wavelength $\lambda=2\pi/k$ suppressed) is exactly equal to $\Pi_B^{(0)}$. Therefore, the smoothed correlation function is given by the classical propagator,

$$V^2\langle|\psi^2(\mathbf{r}_1)\psi^2(\mathbf{r}_2)|\rangle_{\text{smooth}}-1=\Pi_B(\mathbf{r}_1,\mathbf{r}_2), \tag{17}$$

independent of the relation between $|\mathbf{r}_1-\mathbf{r}_2|$ and l_s. The mean free path l_s manifests itself only in setting the scale on which the oscillatory part of $\langle|\psi^2(\mathbf{r}_1)\psi^2(\mathbf{r}_2)|\rangle$ gets damped.

A result for the variance of matrix elements related to Eq. (17) was obtained in Ref. [15] by a semiclassical method. Note, however, that the semiclassical treatment of Ref. [15] is only justified if one introduces a sufficiently large level broadening $\eta\gg\Delta$, while calculating statistical properties of a single eigenfunction requires the limit $\eta\ll\Delta$; see Eq. (6).

Since we have shown that for $l_s\ll L$ the applicability of the Gaussian statistics (1), (3) is restricted to the scales $\ll l_s$, one may be tempted to ask whether increasing l_s beyond L would be favorable from this point of view. The answer is no; in contrast, for $l_s\gtrsim L$ a further increase of l_s reduces the range of applicability of the Gaussian statistics. Indeed, it is not difficult to show that for $l_s\gg L$ the Green's function (9) has the form $\operatorname{Im}G^R(\mathbf{r}_1,\mathbf{r}_2)\simeq-\pi\nu J_0(k|\mathbf{r}_1-\mathbf{r}_2|)$ (we assume for simplicity that the points $\mathbf{r}_1$, $\mathbf{r}_2$ are sufficiently far from the boundary) only for $|\mathbf{r}_1-\mathbf{r}_2|\ll\tilde{l}_s$, where $\tilde{l}_s=L^2/l_s$. At larger distances, $|\mathbf{r}_1-\mathbf{r}_2|\gtrsim\tilde{l}_s$, the Green's function shows irregular oscillations with a characteristic amplitude $|G^R(\mathbf{r}_1,\mathbf{r}_2)|\sim\pi\nu(k\tilde{l}_s)^{-1/2}$ independent of $|\mathbf{r}_1-\mathbf{r}_2|$, which are physically due to the interference of waves multiply reflected from the boundary. Therefore, only at $|\mathbf{r}_1-\mathbf{r}_2|\ll\tilde{l}_s$ the first term in Eq. (12) will dominate and the statistics will be Gaussian.

RANDOM MAGNETIC FIELD

In the above we studied the wave function statistics of a given chaotic system by generating an ensemble of quantum systems with the help of an additional random potential. Now we use the same approach to study the wave function statistics in a random magnetic field (RMF). In this case, an ensemble is defined from the very beginning and the introduction of additional weak disorder may be considered as a technical trick, the reason for which is explained below.

We consider a white-noise RMF $B(\mathbf{r})$ with the correlation function

$$\langle B(\mathbf{r})B(\mathbf{r}')\rangle=\Gamma\,\delta^{(2)}(\mathbf{r}-\mathbf{r}'),\quad \Gamma\ll k^2, \tag{18}$$

and assume that the size of the system, L, is sufficiently large. On length scales longer than the transport mean free path $l_{\text{tr}}=4k/\Gamma$ this problem is described by the conventional unitary-class diffusive σ model [10] so that the results obtained for diffusive systems [8] apply. We will be interested, however, in wave function correlations on much shorter—ballistic—length scales. Specifically, we will study how the Friedel-type oscillations in $\langle|\psi^2(\mathbf{r}_1)\psi^2(\mathbf{r}_2)|\rangle$ decay with increasing $|\mathbf{r}_1-\mathbf{r}_2|$. [The smooth part is simply $V^2\langle|\psi^2(\mathbf{r}_1)\psi^2(\mathbf{r}_2)|\rangle=(\pi k|\mathbf{r}_1-\mathbf{r}_2|)^{-1}$ for all $|\mathbf{r}_1-\mathbf{r}_2|\ll l_{\text{tr}}$, L, as follows from Eq. (17).]

In the case of a random potential the scale for the vanishing of oscillations is set by the single-particle mean free path l_s. An attempt to get an analog of this result by deriving directly the ballistic σ model via averaging over the RMF fails, since the equation for l_s obtained within the self-consistent Born approximation (SCBA) leads to an infrared-divergent and gauge-dependent result [16]. This is a manifestation of the fact that in the case of a RMF the single-particle relaxation rate depends on geometry of the problem.

To overcome this problem, we add an additional weak random potential with the mean free path l_s^{RP} much longer than the length scale of interest set by the RMF (which we will find below). Averaging over this random potential, we derive the σ model in a given realization of the RMF. As explained above, the two-point correlation function of eigenfunction intensities on a scale $|\mathbf{r}_1-\mathbf{r}_2|\ll l_s^{\text{RP}}$ is given by Eqs. (10) and (11). Therefore, the desired oscillatory contribution reads

$$\langle k_q^{\text{osc}}(\mathbf{r}_1,\mathbf{r}_2)\rangle_{\text{RMF}}=-(\pi\nu)^{-2}\operatorname{Re}\langle G^R(\mathbf{r}_1,\mathbf{r}_2)G^R(\mathbf{r}_2,\mathbf{r}_1)\rangle_{\text{RMF}}, \tag{19}$$

where $G^R=(E-\hat{H}+i/2\tau_s^{\text{RP}})^{-1}$ is the Green's function in a given realization of the RMF, and $\langle\cdots\rangle_{\text{RMF}}$ denotes averaging over the RMF realizations. This (second) averaging can be performed with use of the path integral formalism [17]. The product of the two Green's functions in Eq. (19) can be written as

$$\langle G^R(\mathbf{R},0)G^R(0,\mathbf{R})\rangle_{\text{RMF}}=\int_0^\infty dT_1dT_2\int_{\mathbf{r}_i(0)=0}^{\mathbf{r}_i(T_i)=\boldsymbol{R}}\mathcal{D}\mathbf{r}_1\mathcal{D}\mathbf{r}_2 \times\exp[i(E+i/2\tau_s^{\text{RP}})(T_1+T_2)+iS_{\text{kin}}-S_{\text{RMF}}], \tag{20}$$

where $S_{\text{kin}}=\int_0^{T_1}dt\,m\dot{\mathbf{r}}_1^2/2+\int_0^{T_2}dt\,m\dot{\mathbf{r}}_2^2/2$, and $S_{\text{RMF}}=\Gamma s_{\text{no}}/2$, with s_{no} denoting the nonoriented area between the two trajectories $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$. The integral (20) is dominated by the pairs of paths being close to each other and corresponding to an almost uniform and straight motion from 0 to $\mathbf{R}$. To make this explicit, it is useful to perform the change of variables [17], introducing $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$, $\rho=(\mathbf{r}_1+\mathbf{r}_2)/2$, $t_+=(T_1+T_2)/2$, and $t_-=T_2-T_1$ ($t_+\gg t_-$). The RMF-induced part of the action takes then the form

$$S_{\rm RMF}=\frac{v\Gamma}{2}\int_0^{t_+}dt|r^\perp(t)|, \qquad (21)$$

where $v=k/m$ is the particle velocity, and we have split $\mathbf{r}$ into components parallel ($r^\parallel$) and perpendicular ($r^\perp$) to $\dot\rho\approx\mathbf{R}/t_+$. While the integrals over ρ and $r^\parallel$ are essentially the same as for a free particle, that over $r^\perp$ has the form of the Feynman integral for a one-dimensional particle in the potential $i(v\Gamma/2)|x|$. The corresponding Green's function $g(x,x',t)$ reads in the frequency representation at $x=x'=0$ (which is what we need in view of the boundary conditions on $r^\perp$).

$$g(0,0,\omega)=-\frac{i^{-1/3}}{2}(m\tau_0)^{1/2}\frac{\mathrm{Ai}(-i^{-2/3}\omega\tau_0)}{\mathrm{Ai}'(-i^{-2/3}\omega\tau_0)}, \qquad (22)$$

where $\mathrm{Ai}(z)$ is the Airy function, and

$$\tau_0=(4m/\Gamma^2v^2)^{1/3}. \qquad (23)$$

This leads to the following result for the oscillatory part of the wave function correlation function

$$V^2\langle|\psi^2(\mathbf{r}_1)\psi^2(\mathbf{r}_2)|\rangle^{\rm osc}_{\rm RMF}=\frac{1}{\pi kr}\begin{cases}\sin(2kr), & r\ll l_0\\ \sin(2kr+|\zeta_0|r/2l_0-\pi/12) & \\ \times(\pi r/l_0)^{1/2}\exp[-\sqrt{3}|\zeta_0|r/2l_0], & r\gg l_0\end{cases} \qquad (24)$$

where $r=|\mathbf{r}_1-\mathbf{r}_2|$, $\zeta_0\simeq-1.05$ is the lowest zero of $\mathrm{Ai}'(z)$, and $l_0=v\tau_0$.

We thus find that the oscillations are suppressed on the scale $\sim l_0=v\tau_0=(4k/\Gamma^2)^{1/3}$. Note that l_0 is parametrically different from both the transport mean free path $l_{\rm tr}=4k/\Gamma$ and the length $l_{\rm dHvA}=v\tau_{\rm dHvA}=(2\pi/\Gamma)^{1/2}$ characterizing damping of de Haas-van Alphen magnetooscillations of the density of states, $\rho_{\rm osc}\propto\exp[-(\pi/\omega_c\tau_{\rm dHvA})^2]$ [17]. The difference between l_0 and $l_{\rm dHvA}$ (in the case of a random potential, both these scales are set by l_s) illustrates the already mentioned dependence of the single-particle relaxation rate on the geometry of the problem in the case of RMF.

The scale l_0 was obtained in Ref. [18] from consideration of certain Green's function with an obscure physical meaning. We have demonstrated that the length l_0 determines an observable quantity—the scale of decay of the oscillatory part of the wave function correlation function.

CONCLUSIONS

We have studied the wave function statistics in a chaotic ballistic system. The corresponding statistical ensemble is defined by adding a smooth random potential, satisfying $l_{\rm tr}\gg L\gg l_s$. The first inequality preserves the ballistic dynamics, while the second one ensures that the ensemble of quantum systems is sufficiently large and provides meaningful statistics. By using the ballistic σ-model approach we have shown that the conjecture of Gaussian fluctuations of wave functions [3,4] holds on sufficiently short distances $|\mathbf{r}_i-\mathbf{r}_j|\ll l_s$, while it is strongly violated on larger scales. Our results solve, in particular, the problem of inconsistency of the conjecture of Gaussian statistics with the wave function normalization.

We have further applied these results to study the decay of Friedel-type oscillations in the correlation function $\langle|\psi^2(\mathbf{r}_1)\psi^2(\mathbf{r}_2)|\rangle$ in a RMF. In this case averaging over an additional weak random potential yields Gaussian fluctuations of wave functions in a given realization of the RMF. The remaining averaging over the RMF realizations performed via the path integral formalism leads to the result (24). The scale l_0 for the decay of oscillations (playing the role of the single-particle mean free path l_s) is given by Eq. (23), providing physical meaning to a length found in Ref. [18] from some formal consideration.

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